17-2 The Binomial Distribution

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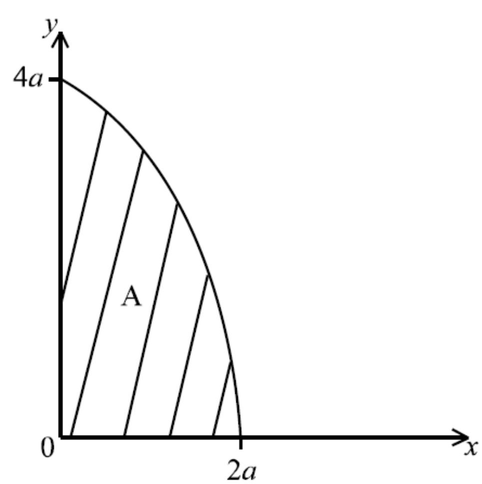
5. *[Maximum mark: 6]*

The diagram below shows the shaded region A which is bounded by the axes and part of the curve $y^2 = 8a(2a - x)$, $a > 0$. Find in terms of a the volume of the solid formed when A is rotated through 360° around the x-axis.



QUESTION 5

$V = \pi \int y^2 \, \mathrm{d}x$ *M1*

$= \pi \int_0^{2a} 8a(2a - x)\, \mathrm{d}x$ *A1A1*

Note: ***A1*** for correct use of y^2, ***A1*** for correct limits.

$= 8\pi a \left[2ax - \frac{x^2}{2} \right]_0^{2a}$ *M1*

$= 8\pi a (4a^2 - 2a^2)$ *(A1)*

$= 16\pi a^3$ *A1* *N0*

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If an experiment has two possible outcomes “success” and “failure”, and their probabilities are p and 1-p respectively, the number of successes (0 or 1) has a Bernoulli Experiment.

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Bernoulli Experiment

A random event is called a Bernoulli Experiment if and only if it has two possible outcomes labeled by x=0 and x=1 in which x=1 (success) occurs with probability p and x=0 (failure) occurs with probability 1-p, where 0<p<1. It therefore has probability function:

$$p(x) = \begin{cases} 1-p & for \quad x=0 \\ p & for \quad x=1 \end{cases}$$

Which can also be written as

$$p(x) = p^x(1-p)^{1-x} \quad for \quad x = 0,1$$

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A Binomial Experiment is one that has the following five characteristics:

The experiment consists of n identical trials

Each trial has 1 out of 2 possible outcomes. We call them success (S) and the other failure (F).

The probability of success of a single trial is p and is constant throughout the whole experiment. The probability of failure is 1-p which is sometimes denotes q where p + q = 1.

The trails are independent.

We are interested in the number of successes x that are possible during n trials. That is x = 0, 1, 2, 3, ….,n.

Basically, a Binomial Experiment is the repetition of a fixed number of independent Bernoulli Experiments and looks at the number of successes in n trials.

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The Binomial Distribution

Suppose that a random experiment can result in two possible mutually exclusive and collectively exhaustive outcomes – success and failure. p is the probability of success on a single trial. If n independent trials are carried out, the distribution of the number of successes x is a Binomial Distribution and its probability distribution function is:

$$p(x) = \binom{n}{x} p^{x}(1-p)^{n-x}$$

for x = 0, 1, 2, 3, …,n

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Ex2. Meet Howard Wallowitz. Howard is a renouned pick up artist. On any given night, Howard approaches 10 women. History has shown that Howard would get 2 phone numbers for every 5 women he approaches.

a.) Is this a binomial experiment? Explain.

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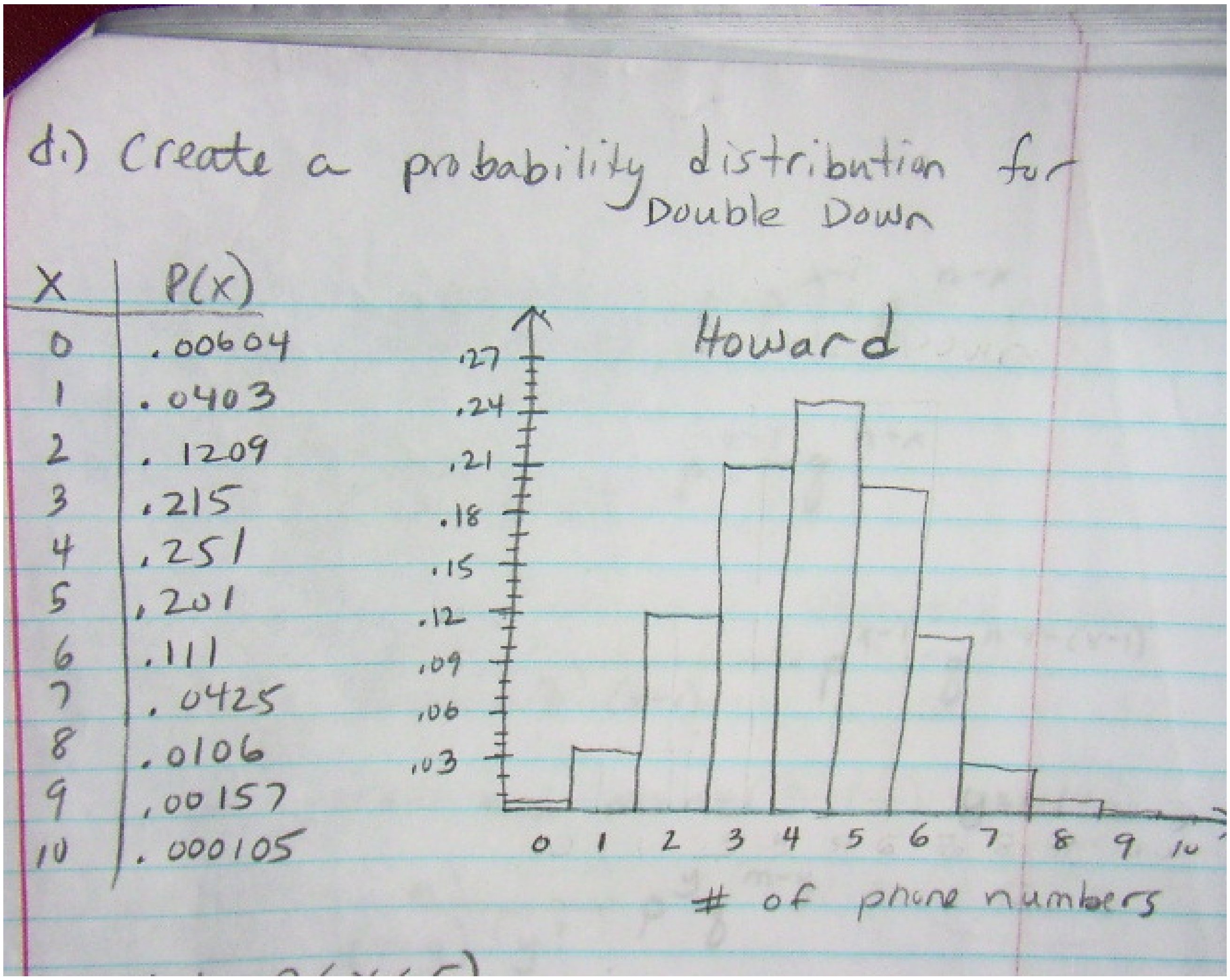
b.) What is the probability of Howard getting 0 phone numbers?

$$P(x=0) = \binom{10}{0} \cdot \left(\frac{2}{5}\right)^0 \cdot \left(\frac{3}{5}\right)^{10}$$

$$= \left(\frac{3}{5}\right)^{10} \approx .00605$$

c.) What is the probability of each of the remaining options?



e.) Find P(x<5)

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The Binomial Probability Model

n = # of trials

p = probability of success

q = 1-p = probability of failure

x = # of successes in n trials

$$p(x) = \binom{n}{x} p^x (1-p)^{n-x}$$

for x = 0, 1, 2, 3, ...,n

Expected Value: $\mu = np$

Variance: $\sigma^2 = npq$

Standard Deviation: $\sigma = \sqrt{npq}$

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f.) Find the expected number of phone numbers that Howard would get in 10 trials.

$EV = \frac{2}{5} \cdot 10 = 4$

g.) Find the variance and standard deviation in the number of phone numbers Howard would get in 10 trials.

$var = \frac{2}{5} \cdot \frac{3}{5} \cdot 10 = 2.4$

$\sigma = \sqrt{2.4} \approx 1.5$

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The Cumulative Binomial Distribution

$$P(X \leq x) = \sum_{y \leq x} P(y)$$

$$= \sum_{y \leq x} \binom{n}{y} p^{y} (1-p)^{n-y}$$

This will give us the probability that X does not exceed the value of x.

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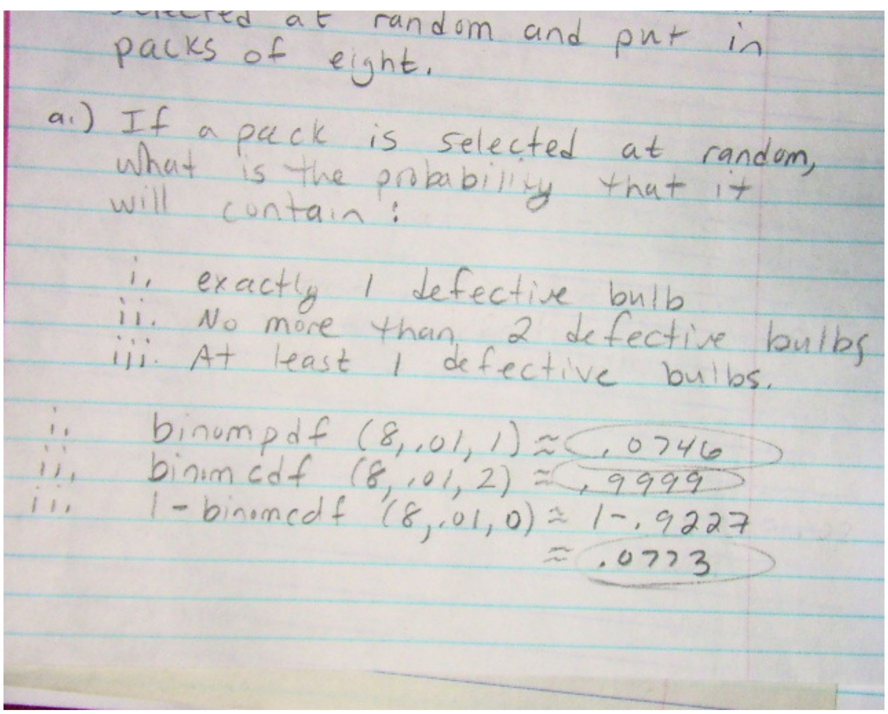
Ex3. In the mass production of the light bulbs, the probability that one bulb is defective is 1%. Bulbs are selected at random and are put in packs of eight.

a.) If a pack is selected at random, what is the probability that it will contain:

i. Exactly 1 defective bulb

ii. Exactly 2 defective bulbs

iii. At least 1 defective bulbs



...lected at random and put in packs of eight.

a.) If a pack is selected at random, what is the probability that it will contain:

i. exactly 1 defective bulb
ii. No more than 2 defective bulbs
iii. At least 1 defective bulbs.

i. binompdf (8,.01,1) ≈ .0746
ii. binomcdf (8,.01,2) ≈ .9999
iii. 1 - binomcdf (8,.01,0) ≈ 1 - .9227
≈ .0773

b.) Given that a pack selected at random contains at least 2 defective bulbs, what is the probability that it contains exactly 3 defective bulbs.

$$P(x=3 / x\geq 2) = \frac{P(x=3 \cap x\geq 2)}{P(x\geq 2)} = \frac{P(x=3)}{P(x\geq 2)}$$

$$= \frac{\text{binompdf}(8, .01, 3)}{1-\text{binomcdf}(8, .01, 1)} = \frac{5.3255 \times 10^{-5}}{.00269008}$$

$$= .0198 \text{ or } 1.98\%$$

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